

# Fluctuating Commutative Geometry

Luiz C. de Albuquerque<sup>1</sup>, Jorge L. deLyra<sup>2</sup>, and Paulo Teotonio-Sobrinho<sup>2</sup>

(1) *Faculdade de Tecnologia de São Paulo - DEG - CEETEPS - UNESP,  
Praça Fernando Prestes, 30, 01124-060 São Paulo, SP, Brazil and*

(2) *Universidade de São Paulo, Instituto de Física - DFMA  
Caixa Postal 66318, 05315-970, São Paulo, SP, Brazil*

We use the framework of noncommutative geometry to define a discrete model for fluctuating geometry. Instead of considering ordinary geometry and its metric fluctuations, we consider generalized geometries where topology and dimension can also fluctuate. The model describes the geometry of spaces with a countable number  $n$  of points. The spectral principle of Connes and Chamseddine is used to define dynamics. We show that this simple model has two phases. The expectation value  $\langle n \rangle$ , the average number of points in the universe, is finite in one phase and diverges in the other. Moreover, the dimension  $\delta$  is a dynamical observable in our model, and plays the role of an order parameter. The computation of  $\langle \delta \rangle$  is discussed and an upper bound is found,  $\langle \delta \rangle < 2$ . We also address another discrete model defined on a fixed  $d = 1$  dimension, where topology fluctuates. We comment on a possible spontaneous localization of topology.

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## I. INTRODUCTION

A possible approach to quantum gravity is to consider it as an Euclidean quantum field theory. In the functional integral approach, one tries to compute a sum over metrics  $g_{\mu\nu}$  defined on a  $d$ -dimensional manifold  $M$ . The dynamics is fixed by a choice of an action that is usually taken to be the Einstein-Hilbert action. An exact computation of such functional integral is not available and several approximated methods have been developed. The main idea is to use some kind of discretization in order to approximate the functional integral by a large number of ordinary integrals or sums. One such method is known as the dynamical triangulation and consist in summing over the triangulations of a given manifold. The situation becomes more complicated when one wishes to include quantum topology change. In this case, the manifold itself is also not fixed and a sum over topologically non equivalent manifolds  $M$  has to be included. The overall picture seems to be under control only for  $d = 2$  where a classification of topologies is possible. Such a classification is proven to be impossible for  $d = 4$  and it is an open problem for  $d = 3$ .

The framework of noncommutative geometry is appropriate to explore some difficult questions in quantum gravity. We illustrate how noncommutative geometry can be used to generalize Euclidean quantum gravity, i.e, fluctuating geometry. Instead of considering ordinary geometry and its metric fluctuations, we consider the generalized geometries where besides the metric, topology and dimension can also fluctuate.

The basic idea coming from noncommutative geometry [1] is that one can describe a Riemannian manifold  $(M, g_{\mu\nu})$  in a purely algebraic way. There is no loss of information if, instead of the data  $(M, g_{\mu\nu})$ , one is given a triple  $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ , where  $\mathcal{A}$  is the C\*-algebra  $C^0(M)$  of smooth functions on  $M$ ,  $\mathcal{H}$  is the Hilbert space of  $L^2$ -spinors on  $M$ , and  $\mathcal{D}$  is the Dirac operator acting on  $\mathcal{H}$ . From the Gelfand-Naimark theorem it is known that the topological space  $M$  can be reconstructed from the set  $\hat{\mathcal{A}}$  of irreducible representations of  $C^0(M)$ . Metric is also encoded, and the geodesic distance can be computed from  $\mathcal{D}$ . Here we will consider only commutative spectral triples - this is enough to go much beyond ordinary geometry. In particular one can treat all Hausdorff topological spaces in this way. Given a pair  $(M, g_{\mu\nu})$ , one can promptly construct the corresponding triple  $(C^0(M), L^2(M), \mathcal{D})$ . However, not all commutative spectral triples, or generalized geometries, come from a pair  $(M, g_{\mu\nu})$ . Nevertheless one can always associate a Hausdorff space  $M = \hat{\mathcal{A}}$  to a commutative spectral triple, where  $\hat{\mathcal{A}}$  denotes the set of irreducible representations of  $\mathcal{A}$ . However, the space  $M$  may not be a manifold. Once we trade the original Riemannian geometry for its corresponding commutative triple we need a replacement for the Einstein-Hilbert action  $S_{EH}$ . The so-called spectral action of Chamseddine and Connes [2] is one possible candidate. It depends only on the eigenvalues of  $\mathcal{D}$  (the spectral principle) and contains  $S_{EH}$  as a dominant term. In this paper however we shall use another spectral action.

The spectral action can be written for any triple, regardless of whether it comes from a manifold  $(M, g_{\mu\nu})$  or not. In the spectral geometry approach it is conceivable to write the partition function

$$Z = \sum_{x \in \mathcal{X}} e^{-S[x]}, \quad (1)$$

where the “sum” is over the set  $\mathcal{X}$  of all possible commutative spectral triples and  $S$  depends on the spectrum of  $\mathcal{D}$ . It includes all Hausdorff spaces and therefore all manifolds of all dimensions.

Apparently, there is no advantage in considering the partition function (1) since it is by no means easier to compute. However, the algebraic approach provides us with a natural way of defining discrete approximations for the theory. For that it is enough to replace the algebra  $\mathcal{A}$  by a finite dimensional algebra  $A_n$ . In this approach to discretization there is no need to introduce a lattice or simplicial decomposition of the underlying space. The approximation of  $\mathcal{A}$  by a finite algebra works

even if the spectral triple does not come from a manifold. In this sense, it gives us a generalization of ordinary discretizations [6]. The simplest discrete model one can consider is a simplification of (1), where instead of summing over the set  $\mathcal{X}$  of commutative spectral triples, we take a subset  $X \subset \mathcal{X}$ . The set  $X$  consists of points  $x = (\mathcal{A}, \mathcal{H}, D)$  where  $\mathcal{A}$  is a commutative algebra with a countable spectrum  $\hat{\mathcal{A}}$ , i.e., with a countable number of irreducible representations. Therefore the underlining space  $M = \hat{\mathcal{A}}$  has a countable and possibly finite number of points.

Most of the results discussed here were reported in [3]. In section 2 we reintroduce the discrete model of [3]. In section 3 we show that this simple model has two phases. The expectation value of the number of points diverges in one phase and it is finite in the other phase. The definition of dimension in noncommutative geometry is recalled in section 4. A estimate of the expectation value of the dimension is discussed in section 5. In section 6 we briefly discuss another model for random geometry where the dimension is fixed and equal to one, but the topology fluctuates. Further results and details on this second model will be reported elsewhere [4].

## II. DISCRETE MODEL

The exact computation of (1) is a major goal, not yet accomplished. In this paper we discretize (1) by sampling the set  $\mathcal{X}$  with finite commutative spectral triples. We will think of it as a useful toy model, which seems to capture some of the main features of the full one, Eq. (1). For instance, the key role played by the eigenvalues of the Dirac (or Laplace) operator in the spectral action approach was emphasized in [7]. In our model they are also the natural dynamical variables due to the connection with random matrix theory. An important ingredient of the model is that the number of points can fluctuate. Moreover, in our simple model the space-time dimension is a dynamical observable and its expectation value can be computable from first principles.

Let us describe the ensemble  $X \subset \mathcal{X}$  of geometries we will consider. A point of  $x \in X$  is a commutative spectral triple  $x = (\mathcal{A}, \mathcal{H}, D)$  where the commutative  $C^*$ -algebra  $\mathcal{A}$  has a countable spectrum  $\hat{\mathcal{A}}$ . We divide  $X$  into subspaces  $X_n$  consisting of triples  $(\mathcal{A}_n, \mathcal{H}_n, D)$  such that  $\hat{\mathcal{A}}_n$  has a fixed number  $n$  of points. From the Gelfand-Naimark theorem it follows that elements of  $\mathcal{A}_n$  are the (possibly infinite) sequences  $a = (a_1, a_2, \dots, a_n)$ ,  $a_j \in \mathbb{C}$ . The Hilbert space  $\mathcal{H}_n$  is given by vectors  $v = (v_1, \dots, v_n)$  with norm  $\|v\|^2 \equiv \sum_{i=1}^n v_i^2 < \infty$ . The elements of  $\mathcal{A}$  are represented by diagonal matrices  $\hat{a} = \text{diag}(a_1, \dots, a_n)$  acting on  $\mathcal{H}_n$ . Finally, the operator  $D$  is a  $n \times n$  self-adjoint matrix. We will sample the space  $X$  by  $X_1, X_2, \dots, X_N$  and eventually take the limit  $N \rightarrow \infty$ .

## III. DYNAMICS

Let  $L$  be a length scale such that the operator  $\mathcal{D}$  given by  $\mathcal{D} = D/L$  will be the analogue of the Dirac operator. The Chamseddine–Connes action depends on a cutoff function of the eigenvalues of  $D/L$ . The cutoff function is zero for eigenvalues of  $D$  greater than  $L$  and one otherwise [2, 7]. In other words, the Boltzmann weight in Eq.(1) would be one outside a compact region in the eigenvalue space, leading to a divergent partition function (see Eq. (11)). Let us consider a quadratic action instead:

$$S[x] = \text{Tr} \left( \frac{\mathcal{D}}{\Lambda} \right)^2 \equiv \beta \text{Tr}(D^2), \quad (2)$$

where  $\Lambda$  is the inverse of Planck's length  $l_p$ , and  $\beta = (l_p/L)^2$ . Finally, we define the partition function  $Z_N(\beta) = \sum_{n=0}^N z_n(\beta)$  where

$$z_n(\beta) = \int [dD] e^{-\beta \text{Tr}(D^2)} \quad (3)$$

is the partition function restricted to  $X_n$ , in other words, an integral over all independent matrix elements  $D_{ij}$ , where  $[dD]$  is the usual measure for  $n \times n$  Hermitian matrices [8]. The partition function  $z_n(\beta)$  defines the one-matrix Gaussian Unitary Ensemble [8]. A straightforward computation gives

$$z_n(\beta) = 2^{\frac{n}{2}} \left( \frac{\pi}{2\beta} \right)^{\frac{n^2}{2}}. \quad (4)$$

The expectation values of an observable  $\mathcal{O}(D_{ij})$  restricted to  $X_n$  and for the entire ensemble are

$$\langle \mathcal{O} \rangle_{n,\beta} \equiv \int [dD] \mathcal{O} e^{-\beta \text{Tr}(D^2)} / z_n(\beta) \quad \text{and} \quad (5)$$

$$\langle \mathcal{O} \rangle(\beta) \equiv \sum_{n=1}^N P(n, \beta) \langle \mathcal{O} \rangle_{n,\beta}, \quad (6)$$

respectively, where the function

$$P(n, \beta) = \frac{z_n(\beta)}{\sum_n z_n(\beta)} \quad (7)$$

is interpreted as the probability of having a universe with  $n$  points. The simplest observable in our model is  $n$ , the number of points in  $\hat{\mathcal{A}}$ . By definition,  $n$  is constant in  $X_n$ , therefore  $\langle n \rangle_{n,\beta} = n$ . Thus we get

$$\langle n \rangle(\beta) = \frac{\sum_n n 2^{\frac{n}{2}} \left( \frac{\pi}{2\beta} \right)^{\frac{n^2}{2}}}{\sum_n 2^{\frac{n}{2}} \left( \frac{\pi}{2\beta} \right)^{\frac{n^2}{2}}}. \quad (8)$$

The mean  $\langle n \rangle$  ("average number of points in the universe") is not a continuous function of  $\beta$  at  $\beta_c = \pi/2$ , signaling the onset of a phase transition. Besides straightforward numerical calculation, there are other ways to show that the sum (8) converges for  $\beta > \beta_c$  and diverges for  $\beta < \beta_c$ .

#### IV. DEFINING DIMENSION $\delta$

For  $\beta < \beta_c$  the relevant universes have  $\langle n \rangle = \infty$  and  $\Delta n / \langle n \rangle = 0$ . For a  $\infty$ -dimensional  $D$  one can define the dimension  $\delta$  of the space  $\hat{\mathcal{A}}$  from the eigenvalues of  $D$ . Let  $\{\mu_0(D), \mu_1(D), \dots\}$  be the modules of the eigenvalues (i.e. the singular values) of  $D$  organized in an increasing order. By the Weyl formula [1], the dimension  $\delta$  is related to the asymptotic behavior of the eigenvalues for large  $k$ :  $\mu_k(D) \approx k^{\frac{1}{\delta}}$ . By definition  $\delta = 0$  for finite dimensional spectral triples. We can argue that  $\langle \delta \rangle$  is of the form

$$\langle \delta \rangle(\beta) = \begin{cases} f(\beta) & \text{if } \beta < \beta_c, \\ 0 & \text{if } \beta > \beta_c. \end{cases} \quad (9)$$

This follows from the fact that for  $\beta > \beta_c$  the probability  $P(n, \beta)$  is localized around some finite  $n$ . Hence  $\langle \delta \rangle$  works as an order parameter. The value  $\beta_c = \pi/2$  separates  $\langle \delta \rangle = 0$  from the rest.

In order to study the dimension we need to consider the spectral  $\zeta$ -function

$$\zeta(z) = \lim_{n \rightarrow \infty} \sum_{k=0}^n \mu_k^{-z} = \text{Tr}(|D|^{-z}), \quad (10)$$

where  $D$  is an  $\infty$ -dimensional matrix ( $\mu_0 > 0$ ). The relation between the dimension and  $\zeta(z)$  has been discussed in [10]. For large enough values of  $\alpha = \text{Re}(z)$ ,  $\text{Tr}(|D|^{-z})$  is well defined. One says that  $D$  has dimension spectrum  $Sd$  if a discrete subset  $Sd = \{s_1, s_2, \dots\} \subset \mathbb{C}$  exists, such that  $\zeta(z)$  can be holomorphically extended to  $\mathbb{C}/Sd$ . This definition is consistent with the Weyl formula. The set  $Sd$  has more than a single point when for example the geometry is the union of pieces of different dimensions [10]. In what follows we will look at an upper bound for the dimension: It may happens that  $\text{Tr}(|D|^{-\alpha}) = 0$  for large enough  $\alpha$ , whereas for small values of  $\alpha$ ,  $\text{Tr}(|D|^{-\alpha}) = \infty$ . Eventually, there is a value of  $\alpha$  (say,  $\alpha_c$ ) for which  $\text{Tr}(|D|^{-\alpha_c})$  is finite and non-zero. The upper bound for the dimension will be  $\delta = \alpha_c$ .

## V. COMPUTING $\langle \delta \rangle$

In order to estimate  $\langle \delta \rangle$  by means of (10), we rewrite (3) and (5) as integrals over the eigenvalues  $\lambda_k$  of  $D$ . The procedure is well-known [8], and leads to  $(C_n \equiv \pi^{\frac{n(n-1)}{2}} / \prod_{k=1}^n k!)$

$$z_n(\beta) = C_n \int_{-\infty}^{\infty} [d^n \lambda] \Delta^2(\lambda_k) e^{-\beta \sum_{i=1}^n \lambda_i^2} \equiv C_n \Psi_{n,\beta}, \quad (11)$$

$$\langle \mathcal{O}(\lambda_i) \rangle_{n,\beta} = \int_{-\infty}^{\infty} [d^n \lambda] \mathcal{O}(\lambda_i) \left\{ \frac{2^{\frac{n(n-1)}{2}} \beta^{\frac{n^2}{2}}}{\pi^{\frac{n}{2}} \prod_{k=1}^n k!} \Delta^2(\lambda_k) e^{-\beta \sum_{i=1}^n \lambda_i^2} \right\} \quad (12)$$

$$\equiv \int_{-\infty}^{\infty} [d^n \lambda] \mathcal{O}(\lambda_i) \mathcal{P}_{n,\beta}(\lambda_k), \quad (13)$$

where  $\Delta(\lambda_k) = \prod_{i < j} (\lambda_j - \lambda_i)$  is the Vandermonde determinant (Jastrow factor), and  $[d^n \lambda] \equiv \prod_{i=1}^n d\lambda_i$ .

In random matrix theory,  $\Psi_{n,\beta}(\gamma)$  is interpreted as the positional partition function of an ensemble of equal charged particles (with positions given by  $\lambda_i$ ) in two dimensions, moving along an infinite line, in thermodynamic equilibrium at temperature  $\gamma$  - the so-called ‘‘Dyson gas’’ [9]. Then,  $\mathcal{P}_{n,\beta}(\lambda_1, \dots, \lambda_n)$  defined in (13) is the probability of finding one particle at  $\lambda_1$ , one at  $\lambda_2$ , etc. The value of  $\Psi_{n,\beta}(\gamma)$  is known from the Selberg’s integral.

In the region  $\beta \leq \beta_c$  the partition function  $Z(\beta)$  is dominated by  $\infty$ -dimensional matrices. Thus, one may try to select the  $\infty$ -dimensional matrices out of the whole ensemble, and then compute the mean of the  $\zeta$ -functions following (10). However, from the standpoint of our statistical approach this procedure does not seems natural since the sum over  $n$  is a key ingredient in the whole construction. Hence, we look for a quantity related to the  $\zeta$ -function that captures the statistical nature of our model. Let us compute the mean value

$$\langle \text{Tr}_{\kappa} |D|^{-\alpha} \rangle_{n,\beta} = \left\langle \sum_{k=1}^n |\lambda_k|^{-\alpha} \theta(|\lambda_k| - \kappa) \right\rangle_{n,\beta}. \quad (14)$$

After some considerations one arrives at [3]

$$\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-\alpha} \rangle_{n,\beta} \approx \frac{2}{\pi} (2n)^{1-\frac{\alpha}{2}} \beta^{\frac{\alpha}{2}} \int_{\epsilon}^1 \frac{dy}{y^{\alpha}} \sqrt{1-y^2}. \quad (15)$$

The asymptotic behavior of  $\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-\alpha} \rangle_{n,\beta}$  does not depend in an essential way on the particular choice of  $\epsilon$ , as long as we keep  $\epsilon \neq 0$ .

Now we use the asymptotic formula (15) and search for the value  $\alpha_c$  for which, as  $N \rightarrow \infty$  and  $\beta \rightarrow \beta_c$ ,  $\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-\alpha} \rangle$  diverges (converges to zero) if  $\alpha < \alpha_c$  ( $\alpha > \alpha_c$ ), with  $\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-\alpha_c} \rangle$  finite and non-zero. This gives an upper bound for the dimension of the “condensed” manifold in the infinite phase ( $\beta \leq \beta_c$ ), which is  $\langle \delta \rangle < \alpha_c$ . We obtain:

$$\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-\alpha} \rangle(\beta) \sim \lim_{N \rightarrow \infty} \sum_{n=1}^N P(n, \beta) n^{1-\frac{\alpha}{2}}. \quad (16)$$

In the finite phase ( $\beta > \beta_c$ ) the sum in (16) converges for  $\alpha \geq 0$ . We conclude that  $\alpha_c = 0$  (i.e.  $\langle \delta \rangle = 0$ ) for  $\beta > \beta_c$ , as expected. From the behavior of  $P(n, \beta)$  in the infinite phase it follows that the convergence of the sum in (16) is dictated by the behavior of  $\Gamma_{n,\alpha} = n^{1-\frac{\alpha}{2}}$  in the limit  $n \sim N \rightarrow \infty$ . For  $\beta \leq \beta_c$  we get  $\Gamma_{n,\alpha} \rightarrow \infty$  if  $\alpha < 2$ , and  $\Gamma_{n,\alpha} \rightarrow 0$  if  $\alpha > 2$ . For  $\alpha = 2$  it turns out that  $\langle \text{Tr}_{\kappa_{n,\beta}} |D|^{-2} \rangle(\beta) \sim 1$ . Therefore, we obtain the upper bound  $\langle \delta \rangle < 2$ .

## VI. ONE DIMENSIONAL MODEL

We would like to apply the ideas of last sections to spaces of dimension one instead of dimension zero. Let us consider a collection  $X$  of  $n$  one dimensional intervals. The corresponding spectral triple will be  $(A_X, \mathcal{H}_X, D)$  where  $A_X$  is the algebra of continuous functions on  $X$  and  $\mathcal{H}_X = L^2(X)$ . The analogue of the Dirac operator will be the momentum operator  $-i\frac{\partial}{\partial x}$ . We will keep  $X$  fixed and fluctuate  $D$ .

Let us consider a simple example where  $X$  is a pair of disjoint intervals  $I_1, I_2$ . The intervals will be parametrized by coordinate  $x \in [0, 2\pi]$ . An element  $\psi \in \mathcal{H}_X$  is a pair of functions  $\psi_1(x), \psi_2(x)$ ,  $\psi_i : I_i \rightarrow \mathbb{C}$  and the scalar product is

$$(\psi, \chi) = \int_0^{2\pi} dx (\psi_1^* \chi_1 + \psi_2^* \chi_2) \quad (17)$$

We can write  $\psi$  as a column vector and the operator  $D$  as the following matrix

$$D = \begin{pmatrix} -i\partial_x & 0 \\ 0 & -i\partial_x \end{pmatrix} \quad (18)$$

We have not fixed completely the spectral triple. The operator  $D$  is fixed only up to boundary conditions (BC's) or self-adjoint extensions. The most general BC can be written as

$$\begin{pmatrix} \psi_1(2\pi) \\ \psi_2(2\pi) \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} \psi_1(0) \\ \psi_2(0) \end{pmatrix}, \quad (19)$$

where  $g$  is a matrix in  $U(2)$ .

We may ask what geometrical properties of  $X$  is determined by such BC's. The point of view taken by Balachandran at all in [5] is that a BC fixes the global topology. A couple of examples will illustrate their point of view. First let us suppose  $g_{ij} = \delta_{ij}$ . In this case  $\psi_i(0) = \psi_i(2\pi)$ . The two intervals fold into a pair of independent circles. Now suppose  $g_{11} = g_{22} = 0$  and  $g_{12} = g_{21} = 1$ . Therefore  $\psi_1(2\pi) = \psi_2(0)$  and  $\psi_2(2\pi) = \psi_1(0)$ . As one can see, the two intervals are connected to make a single circle of size  $4\pi$ . For a generic BC, however, topology is not localized as in these two examples but it is rather a superposition of both. We refer to [5] for further details.

We would like to compute the partition function

$$Z_X(\beta) = \sum_D e^{-\beta \text{Tr} D^2} \quad (20)$$

and look at the probability distribution for the BC's. For finite values of  $\beta$  the topology, as given by the BC, will fluctuate. We have evidence, however, that the BC gets localized as we increase  $\beta$  and becomes a pair of circles in the infinite limit. The significance of this fact and generalizations of this one dimensional model will appear in [4].

## VII. FINAL REMARKS

We proposed a discrete model for Euclidean quantum gravity, i.e, random geometry, based on the framework of noncommutative geometry. The first model contains the mean number of points,  $\langle n \rangle$ , and the dimension of the space-time,  $\langle \delta \rangle$ , as dynamical observables. We have shown that the discrete model has two phases: a finite phase with a finite value of  $\langle n \rangle$  and  $\langle \delta \rangle = 0$ , and an infinite phase with a diverging  $\langle n \rangle$  and a finite  $\langle \delta \rangle \neq 0$ . An upper bound for the order parameter  $\langle \delta \rangle$  was found,  $\langle \delta \rangle < 2$ . We also considered the simplest example of another model where dimension is fixed but topology fluctuates. In the limit of infinite  $\beta$ , however, topology gets localized .

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